

# MATHEMATICS

## Chapter 14: Symmetry



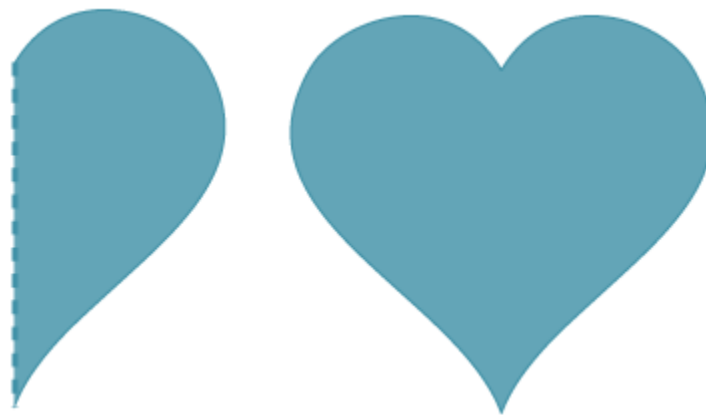
# Symmetry

## Basics of Symmetry

- Symmetry is a geometrical concept that is found in most cases including nature.
- Any geometric shape can be said to be symmetric or asymmetric
- A shape is said to be symmetric if there exists an imaginary line passing through that divides the shape into halves and that these halves overlap each other completely.
- In other words, fold the shape about the imaginary line to check if the two halves completely overlap each other or not. If they overlap each other completely the shape is symmetric, if not, then it is asymmetric.
- The imaginary line is called as the line of symmetry.
- The symmetry observed in the above example is called as a line or bilateral symmetry.

## Symmetry in Mathematics

In Mathematics, a meaning of symmetry defines that one shape is exactly like the other shape when it is moved, rotated, or flipped. Consider an example, when you are told to cut out a 'heart' from a piece of paper, don't you simply fold the paper, draw one-half of the heart at the fold and cut it out to find that the other half exactly matches the first half? The heart carved out is an example of symmetry.

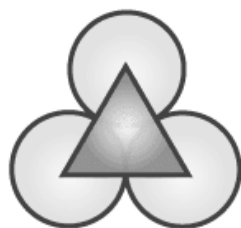


Symmetry Math definition states that “symmetry is a mirror image”. When an image looks identical to the original image after the shape is being turned or flipped, then it is called symmetry. It exists in patterns. You may have often heard of the term ‘symmetry’ in day to day life. It is a balanced and proportionate similarity found in two halves of an object, that is, one-half is the mirror image of the other half. And a shape that is not symmetrical is referred to as asymmetrical. Symmetric objects are found all around us, in nature, architecture, and art.

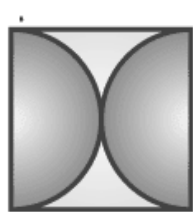
## Symmetrical Figures

Symmetrical shapes or figures are the objects where we can place a line such that the

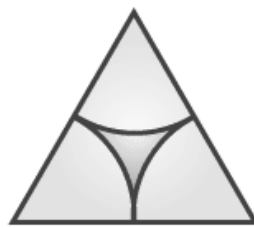
images on both sides of the line mirror each other. The below set of figures form symmetrical shapes when we place a plane or draw the lines.



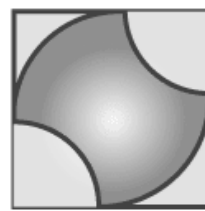
(a)



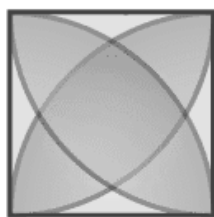
(b)



(c)



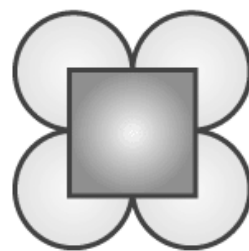
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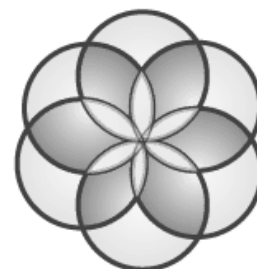
(e)



(f)



(g)



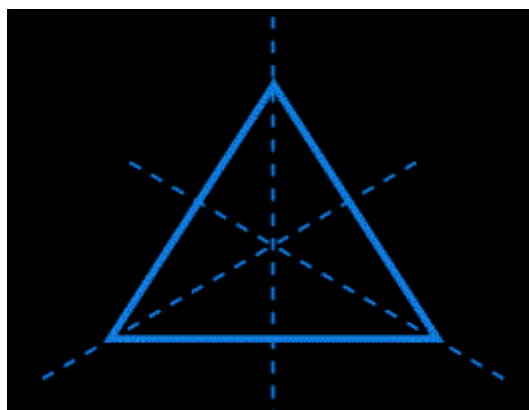
(h)

## Reflection Symmetry

- There exists at least one line that divides a figure into two halves such that one-half is the mirror image of the other half.
- Reflection symmetry is a unique case of line symmetry as there exists lateral inversion in the two halves.
- Lateral inversion signifies that left side of one half is the right side of its mirror half.

## Examples of Reflection Symmetry

Regular polygons of  $N$  sides have  $N$  lines of symmetry.

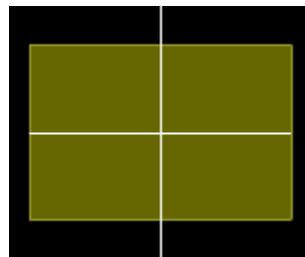


Square: 4 Lines of Symmetry



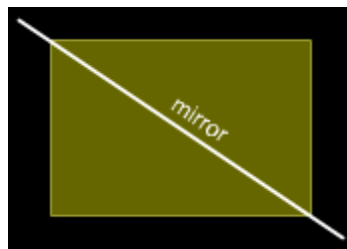
Pentagon: 5 Lines of Symmetry

A rectangle has a vertical and a horizontal line of symmetry.



Rectangle: 2 Lines of Symmetry

However, it is not symmetrical across its diagonal.



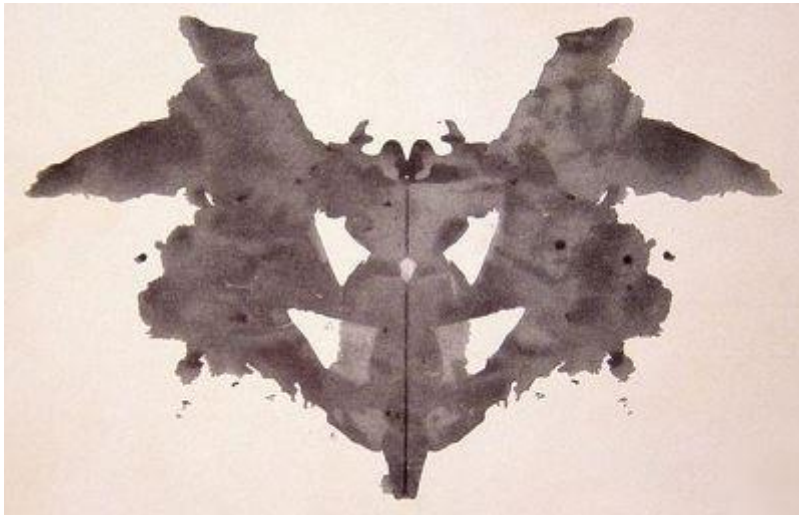
## Is Parallelogram Symmetrical?

At first glance, a parallelogram may look symmetrical. But it is not symmetrical along any of the axes of reflection.



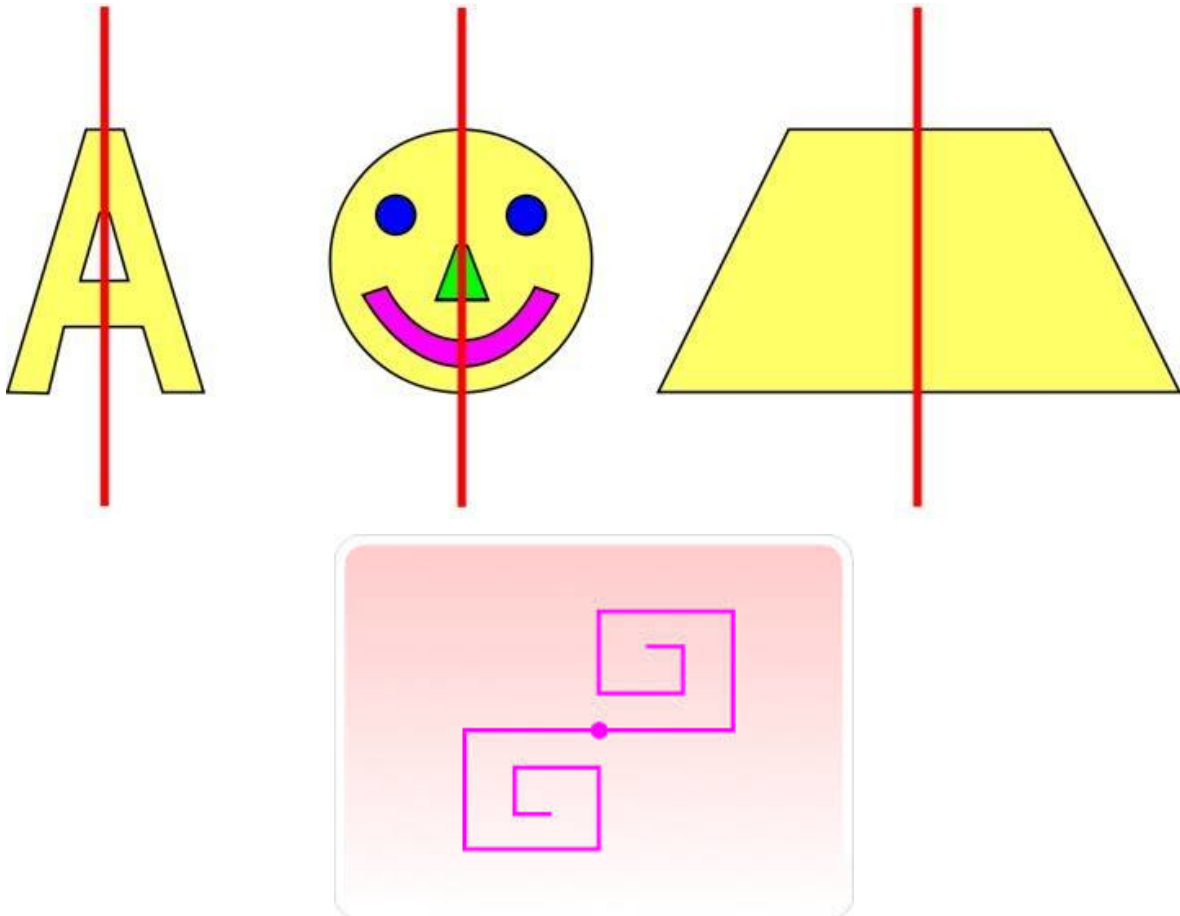
Reflection symmetry of Parallelogram

Reflection symmetry is also observed in inkblot paper. Pour a little ink and water drop on one side of a paper. Fold the paper into two halves and press it with your palm. Unfold the paper to see the symmetric pattern along the line of the fold. The line of the fold is the axis of reflection.



### Point Symmetry

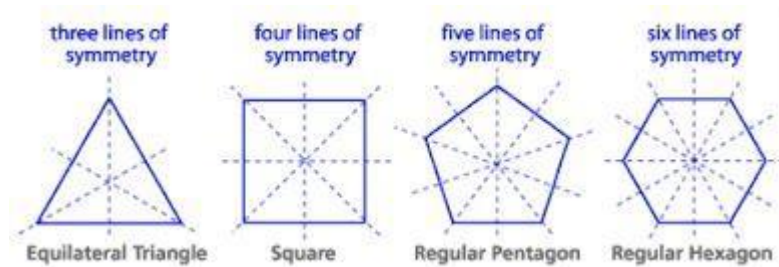
If a shape has point symmetry, then any point on the shape has a matching point which is exactly at the same distance from the point of symmetry but in the opposite direction.



### Lines of symmetry for regular polygons

- Regular polygons are closed shapes that have equal sides and equal angles.
- Such polygons have multiple lines of symmetry.

Some regular polygons with their line of symmetry are shown below:



## Rotational Symmetry

- There exists another concept of symmetry that does not involve folding the figure to check if they coincide.
- Rotational symmetry is one where when a figure is rotated about a centre point, the figure looks exactly the same before rotation.
- The particular centre for which rotational symmetry is observed is called the centre of rotation.
- The angle of turning during rotation is called angle of rotation.
- The number of positions in which a figure can be rotated and still appears exactly like it did before the rotation is called order of symmetry.

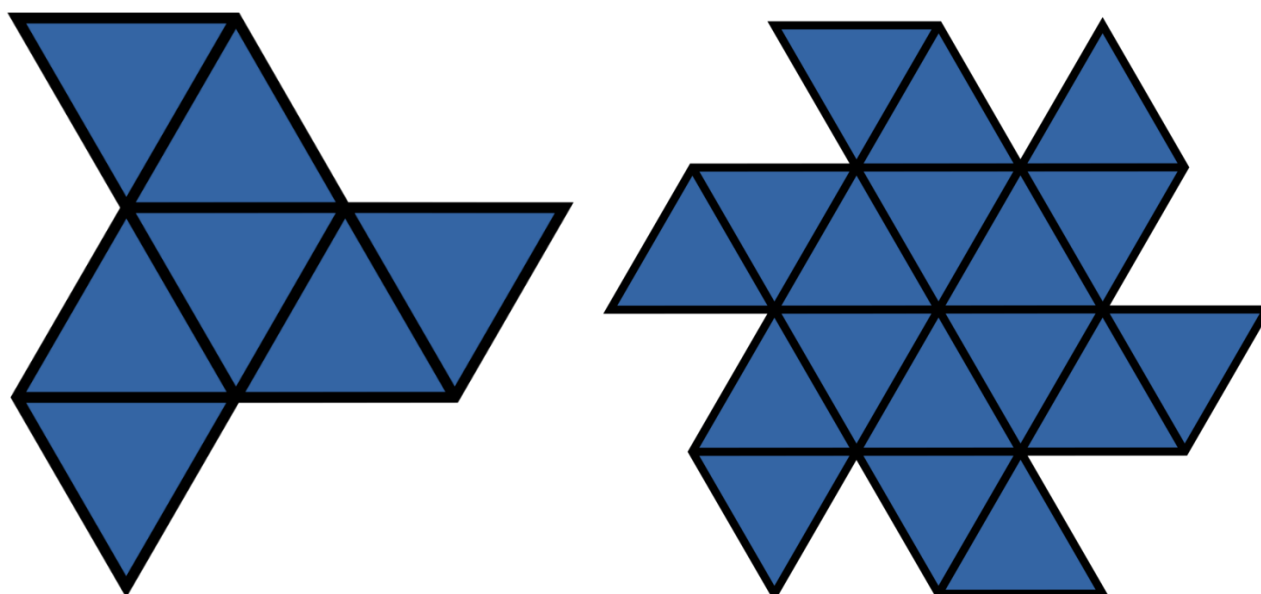
**Rotational symmetry:** A figure is rotated around a center point and it still appears exactly like it did before the rotation

**Centre of rotation:** Fixed point around which the rotation occurs

**Angle of rotation:** Angle of turning during rotation

**Order of Symmetry:** Number of positions in which a figure can be rotated and still appears exactly like it did before the rotation

**Example:** A star can be rotated 5 times along its tip and looks same every time. Hence, its order of symmetry is 5.



**Center of Rotation**

For a figure or object that has rotational symmetry, the fixed point around which the rotation occurs is called the centre of rotation. Example: the centre of rotation of a windmill in the centre of the windmill from which its blades originate.

### Angle of Rotational Symmetry

For a figure or object that has rotational symmetry, the angle of turning during rotation is called the angle of rotation. Example: when a square is rotated by 90 degrees, it appears the same after rotation. So, the angle of rotation for a square is 90 degrees.

In the same way, a regular hexagon has an angle of symmetry as 60 degrees, a regular pentagon has 72 degrees, and so on.

### Order of Rotational Symmetry

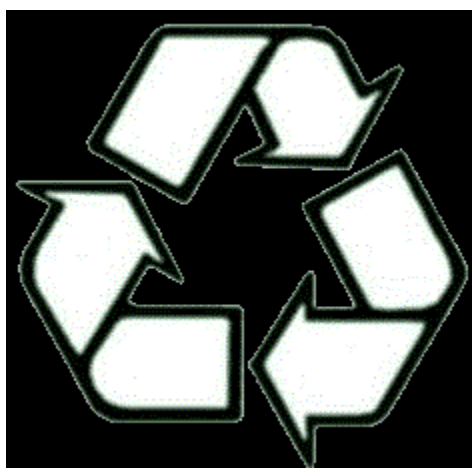
The number of positions in which a figure can be rotated and still appears exactly as it did before the rotation, is called the order of symmetry. For example, a star can be rotated 5 times along its tip and look at the same every time. Hence, its order of symmetry is 5.

### Rotational Symmetry Letters

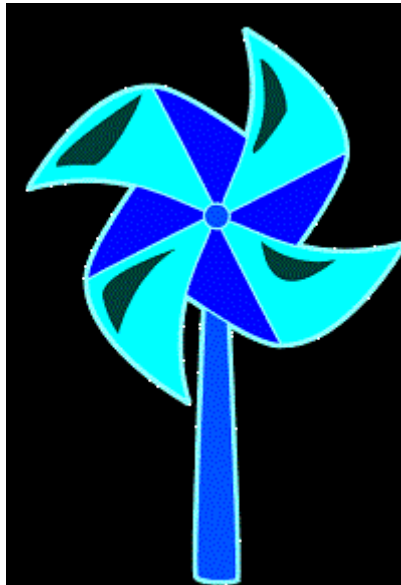
There are many capital letters of English alphabets which has symmetry when they are rotated clockwise or anticlockwise about an axis. Some of them are: Z, H, S, N and O. When these letters are rotated 180 degrees clockwise or anticlockwise the letters appears to be same.

### Examples of Rotational Symmetry

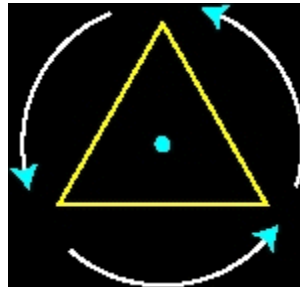
The recycle logo has an order of symmetry of 3.



The paper windmill has an order of symmetry of 4.



The triangle has an order of symmetry of 3.



The Swastik symbol has an order of symmetry of 4.



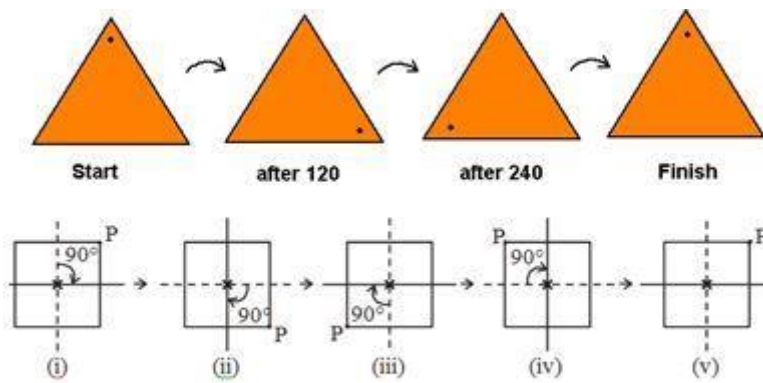
The roundabout road sign has an order of symmetry of 3.



## Geometry in rotational symmetry

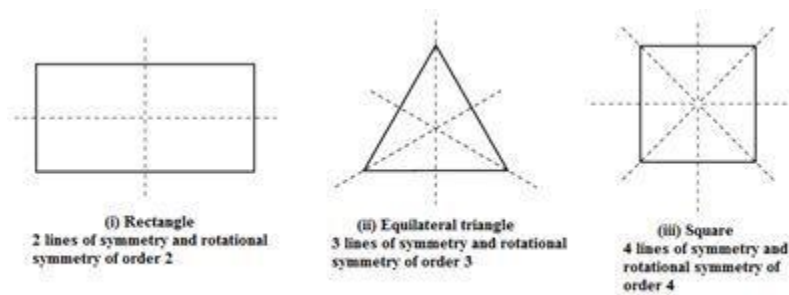
Rotational symmetry in triangle and square:





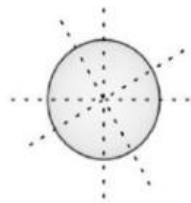
## Summary of Symmetry

Figures having both line symmetry and rotational symmetry:

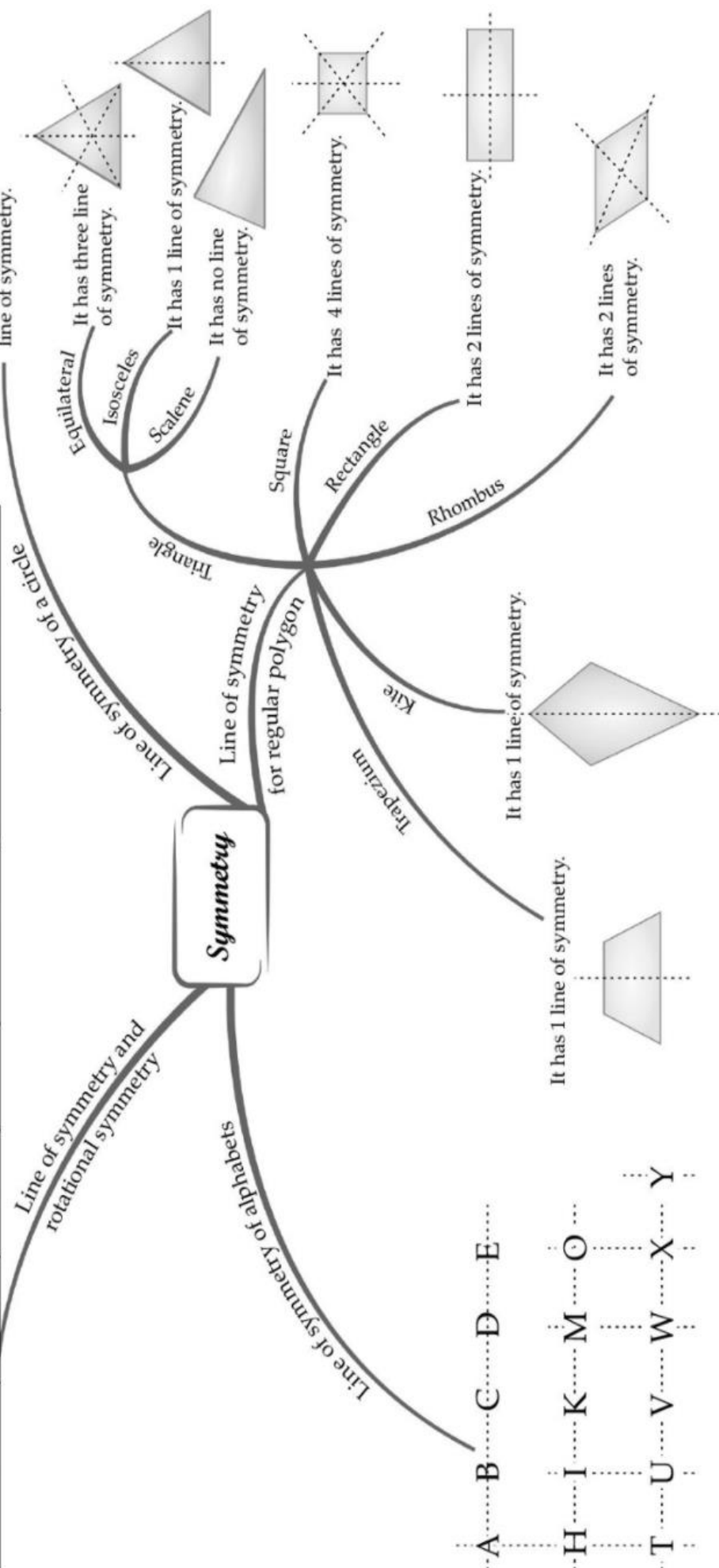


# CHAPTER - 14 SYMMETRY

Figure names	Line symmetry	Number of symmetry	Rational symmetry	Centre of rotation	Order of rotational symmetry
Square	Yes	4	Yes	Point of intersection of diagonals	4
Rectangle	Yes	2	Yes	Intersection of diagonals	2
Equilateral triangle	Yes	3	Yes	Centroid	3
Regular hexagon	Yes	6	Yes	Centre of hexagon	6
Circle	Yes	Infinite	Yes	Centre	Infinite
Parallelogram	No	0	Yes	Intersection of diagonals	2
Rhombus	Yes	2	Yes	Intersection of diagonals	2



A circle has many diameters. All diameters of circle are its line of symmetry.



## Important Questions

### Multiple Choice Questions-

Question 1. How many lines of symmetry are there in an equilateral triangle?

- (a) 1
- (b) 2
- (c) 3
- (d) 4

Question 2. How many lines of symmetry are there in a square?

- (a) 1
- (b) 2
- (c) 3
- (d) 4

Question 3. How many lines of symmetry are there in a rectangle?

- (a) 1
- (b) 2
- (c) 3
- (d) 4

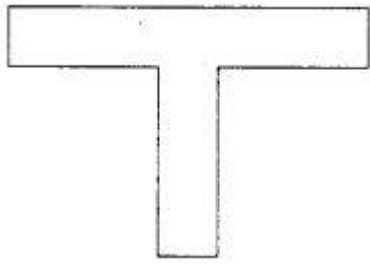
Question 4. How many lines of symmetry are there in a regular pentagon?

- (a) 1
- (b) 2
- (c) 3
- (d) 5

Question 5. How many lines of symmetry are there in a regular hexagon?

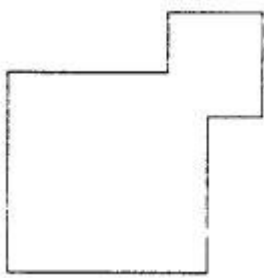
- (a) 2
- (b) 4
- (c) 6
- (d) 3

Question 6. How many lines of symmetry are there in the following figure?



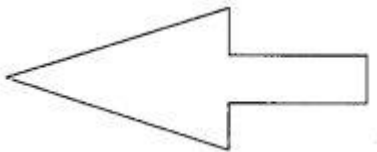
- (a) 1
- (b) 2
- (c) 3
- (d) 4

Question 7. How many lines of symmetry are there in the following figure?



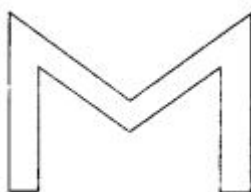
- (a) 1
- (b) 2
- (c) 3
- (d) 4

Question 8. How many lines of symmetry are there in the following figure?



- (a) 1
- (b) 2
- (c) 3
- (d) 4

Question 9. How many lines of symmetry are there in the following figure?



- (a) 4

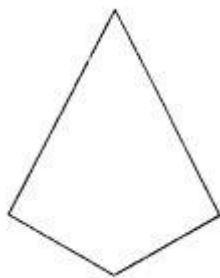
- (b) 3
- (c) 2
- (d) 1

Question 10. How many lines of symmetry are there in the following figure?



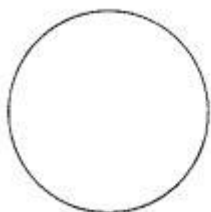
- (a) 2
- (b) 1
- (c) 4
- (d) 3

Question 11. How many lines of symmetry are there in following figure?



- (a) 1
- (b) 2
- (c) 3
- (d) None of these

Question 12. How many lines of symmetry are there in the following figure?



- (a) 1
- (b) 2
- (c) 3
- (d) Infinitely many

Question 13. How many lines of symmetry are there in an isosceles triangle?

- (a) 4
- (b) 3
- (c) 1
- (d) 2

Question 14. How many lines of symmetry are there in a scalene triangle?

- (a) 1
- (b) 0
- (c) 2
- (d) 4

Question 15. How many lines of symmetry are there in a rhombus?

- (a) 1
- (b) 2
- (c) 3
- (d) 4

### Very Short Questions:

1. Draw any two English alphabets having an only a vertical line of symmetry.
2. Draw any two English alphabets having a horizontal line of symmetry.
3. Draw any two English alphabets having both horizontal and vertical line of symmetry.
4. Draw any two English alphabets which has no line of symmetry.
5. Draw any two figures which have the order of rotational symmetry 4.
6. Draw any two figures which have the order of rotational symmetry 2.
7. State the order of rotational symmetry of the following figures.



Equilateral triangle

(i)



Regular pentagon

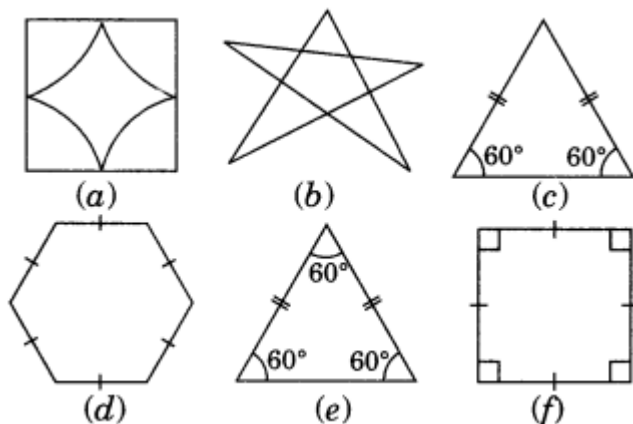
(ii)

8. Draw a figure having an infinite number of lines of symmetry.
9. Draw any two figure having no lines of symmetry.
10. State the English alphabet which has only the horizontal line of

symmetry.

### Short Questions :

1. Give the order of rotational symmetry of each of the following figures:



2. How many lines of symmetry do the following have:

- (a) a parallelogram
- (b) an equilateral triangle
- (c) a right angle with equal legs
- (d) an angle with equal arms
- (e) a semicircle
- (f) a rhombus
- (g) a square
- (h) scalene triangle

3. What letters of the English alphabet have reflectional symmetry about?

- (a) a vertical mirror
- (b) a horizontal mirror
- (c) both horizontal and vertical mirrors

### Long Questions :

1. Give three examples of shapes with no line of symmetry.

### Answer Key-

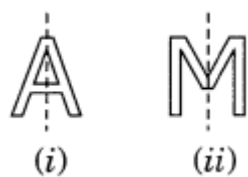
### Multiple Choice questions-

1. (c) 3
2. (d) 4
3. (b) 2
4. (d) 5

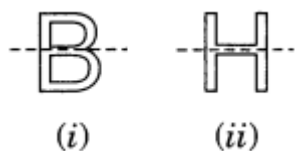
5. (c) 6
6. (a) 1
7. (a) 1
8. (a) 1
9. (d) 1
10. (b) 1
11. (a) 1
12. (d) Infinitely many
13. (c) 1
14. (b) 0
15. (b) 2

### Very Short Answer :

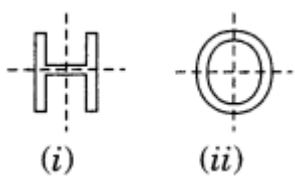
1.



2.



3.

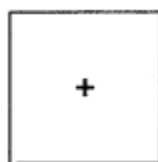


4.

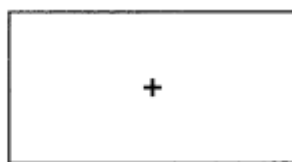


5.



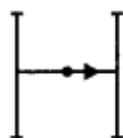


Square  
(i)



Rectangle  
(ii)

6.



English alphabet  
H



English alphabet  
S

7. (i) Order of equilateral triangle = 3  
(ii) Order of regular pentagon = 5.

8.



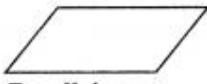
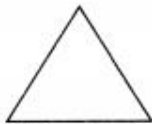
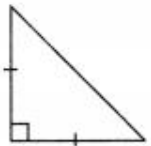
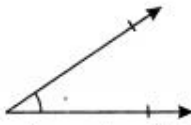

Circle

A circle has the infinite number of lines of symmetry.

9. English alphabet R and P have no lines of symmetry.  
10. B, C, D, E, H, I, O, X and K are the English alphabets having an only horizontal line of symmetry.

### Short Answer :

1. (a) Order of rotational symmetry = 4  
(b) Order of rotational symmetry = 5  
(c) Order of rotational symmetry = 3  
(d) Order of rotational symmetry = 6  
(e) Order of rotational symmetry = 3  
(f) Order of rotational symmetry = 4
- 2.

- (a)  No line of symmetry  
Parallelogram
- (b)  Number of lines of symmetry = 3  
Equilateral triangle
- (c)  Number of lines of symmetry = 1  
Right angled isosceles triangle
- (d)  Number of lines of symmetry = 1  
An angle with equal arm
- (e)  Number of lines of symmetry = 1  
A semicircle

3. (a) The letters of English alphabet having reflectional symmetry about a vertical mirror are:  
A, H, I, M, O, T, U, V, W, X, Y
- (b) Letters of the English alphabets having reflectional symmetry about a horizontal mirror are:  
B, C, D, E, H, I, O, X
- (c) Letters of the English alphabet having reflectional of symmetry about vertical and horizontal mirror are:  
O, X, I, H.

### Long Answer :

1. A shape has a no line of symmetry, if there is no line about which the figure may be folded and also parts of the figure will not coincide.  
A scalene triangle, a quadrilateral and a parallelogram

